

# Synthetic Lorentz force in classical atomic gases via Doppler effect and radiation pressure

T. Dubček,<sup>1</sup> N. Šantić,<sup>1</sup> D. Jukić,<sup>1,2</sup> D. Aumiler,<sup>3</sup> T. Ban,<sup>3</sup> and H. Buljan<sup>1,\*</sup>

<sup>1</sup>*Department of Physics, University of Zagreb, Bijenička c. 32, 10000 Zagreb, Croatia*

<sup>2</sup>*Max Planck Institute for the Physics of Complex Systems,  
Nöthnitzer Strasse 38, 01187 Dresden, Germany*

<sup>3</sup>*Institute of Physics, Bijenička c. 46, HR-10000 Zagreb, Croatia*

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We theoretically predict a novel type of synthetic Lorentz force for classical (cold) atomic gases, which is based on the Doppler effect and radiation pressure. A fairly uniform and strong force can be constructed for gases in macroscopic volumes of several cubic millimeters and more. This opens the possibility to mimic classical charged gases in magnetic fields, such as those in a tokamak, in cold atom experiments.

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The quest for synthetic magnetism in quantum degenerate atomic gases is motivated by producing controllable quantum emulators, which could mimic complex quantum systems such as interacting electrons in magnetic fields [1]. An appealing idea is to place the atomic gas in a specially tailored laser field which, due to laser-atom interactions, acts as a synthetic magnetic field for neutral atoms [2]. The mechanism is based on the analogy between the Aharonov-Bohm phase accumulated when a charged quantum particle undergoes a closed loop in a magnetic field, and the Berry phase accumulated when an atom adiabatically traverses a closed loop in the tailored laser field [2, 3]. Recent experiments in bulk Bose-Einstein condensates (BECs) have produced synthetic magnetic fields by spatially dependent optical coupling between the internal states of the atoms [4, 5]. Superfluid vortices [4] and the Hall effect [5] were observed as signatures of synthetic magnetism in those BECs. Synthetic magnetism in optical lattices is achieved by engineering the complex tunneling parameter between the lattice sites, which is experimentally accomplished by different means [6, 7]. Interestingly, even Dirac monopoles were observed in a synthetic magnetic field produced by a spinor BEC [8]. Synthetic magnetic fields for light (e.g., see [9]) are also attractive; recently they were observed in deformed honeycomb photonic lattices [10]. However, *classical* (rather than quantum degenerate) cold atomic gases have been circumvented in the quest for synthetic magnetism, even though they could emulate in a controllable fashion, and in table-top experiments, versatile complex classical systems [11, 12]; one desirable system for table-top emulation is tokamak plasma. Laser forces on atoms in classical gases depend on atomic velocity [13] and position [14]. A typical example is the Doppler cooling force - a viscous damping force that cools a classical gas to  $\mu\text{K}$  temperatures [13, 14]. Here we demonstrate a novel scheme for creating synthetic Lorentz force via the Doppler effect and radiation pressure, which is applicable for classical cold atomic gases. The experimental realiza-

tion of the scheme is proposed with  $^{87}\text{Rb}$  atoms cooled in a Magneto-Optical Trap (MOT). The signature of the Lorentz force can be observed in motion of the center of mass (CM) and/or the shape of the atomic cloud.

Numerous schemes have been proposed to create synthetic magnetic fields with ultracold atoms (see [1, 2, 15] for reviews). In the approach based on the Berry phase [3], when atoms move in space, they adiabatically follow the ground state of the light-atom coupling (dressed state), which depends on the spatial coordinates [2, 3]. Their CM wavefunction acquires a geometric (Berry) phase, which corresponds to the gauge potentials [2]. The synthetic magnetic (and electric [16]) fields are derived from these gauge potentials [2]. In these schemes spontaneous emission must be minimized to prevent heating of the ultracold gas. For this reason, the dressed (ground) state is often a superposition of quasidegenerate ground states [3, 17, 18], i.e., the population of excited states is negligible [17, 18]. A semi-classical interpretation of geometric gauge potentials, i.e., the connection with the Lorentz force was reported in Ref. [19].

Another avenue for creating artificial magnetic fields in ultracold atomic gases is to rotate the system at some angular frequency, which provides the synthetic Lorentz force in the rotating frame [15]; the role of the Lorentz force is played by the Coriolis force. This scheme is suitable for rotationally invariant trapping potentials. However, the laser-atom interactions avenue is more appealing since it does not impose symmetries and produces synthetic magnetic fields in the laboratory frame [2].

Any scheme for synthetic magnetism in classical atomic gases must be operational on atoms moving with fairly large velocities (at least up to  $\sim 0.5\text{m/s}$ ). The Berry phase method demanding adiabatic dynamics is therefore limited [2]. On the other hand, schemes for classical gases do not need to be limited by avoiding spontaneous emission. Next, classical gases in a standard MOT are typically of millimeter size [14] and therefore the synthetic Lorentz force should be large in volumes of at least

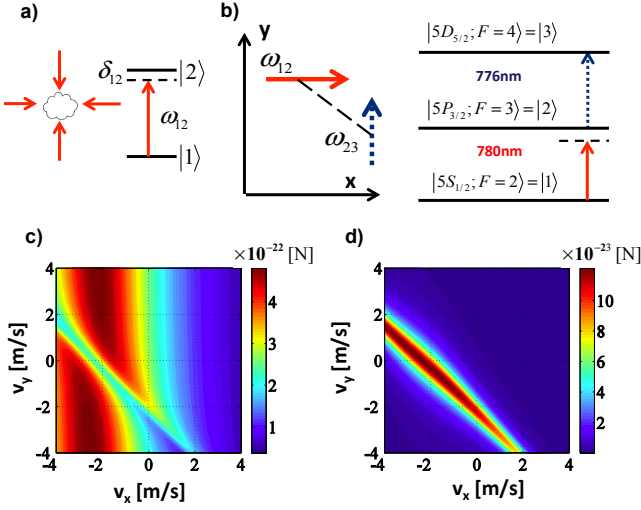


FIG. 1. Sketch of the main idea for constructing the synthetic Lorentz force. (a) Illustration of the setup for the standard Doppler cooling force (using two-level atoms). (b) The idea for the synthetic Lorentz force in the simplest three-level system that can be realized with  $^{87}\text{Rb}$  atoms. Dashed line indicates that two-step absorption of  $\omega_{12} + \omega_{23}$  yields  $F_y$ . The force components  $F_x$  (c) and  $F_y$  (d) calculated as a function of the atomic velocity. See text for details.

a few cubic millimeters. With these guidelines in mind, it seems prosperous to seek for a novel scheme using laser-atom interactions for creating synthetic Lorentz forces in classical atomic gases.

The scheme proposed here is based on the Doppler effect and radiation pressure. The standard Doppler cooling force arises when a laser field is red-detuned compared to the atomic resonance frequency as sketched in Fig. 1(a) [13, 14]. Due to the Doppler effect, the atom has greater probability for absorbing a photon when it moves towards the light source. Absorption changes atom's momentum along the laser propagation axis, whereas spontaneously emitted photons yield random kicks. Cycles of absorption and emission result in a viscous damping force  $\mathbf{F}_D(\mathbf{v}) \approx -\alpha\mathbf{v}$  for small velocities [14]; it is collinear with the velocity and is used to obtain optical molasses [14].

Our first objective is to construct a laser-atom system (in the  $xy$  plane) where  $F_y$  depends on  $v_x$ . To achieve this via Doppler effect we utilize multilevel structure of the atoms. The simplest scheme is sketched in Fig. 1(b) where a three-level atom interacts with two orthogonal laser beams (linearly polarized along  $z$ ). The laser  $\omega_{12}$  is red detuned:  $\delta_{12} = \omega_{12} - (E_2 - E_1)\hbar^{-1} < 0$ , whereas  $\omega_{23}$  is on resonance:  $\delta_{23} = 0$ . The absorption of  $\omega_{23}$  photons, which results in  $F_y$ , is the second step in the two-step two-photon absorption process:  $|1\rangle \rightarrow |2\rangle \rightarrow |3\rangle$ . The probability for two-step absorption depends on the Doppler shifted detuning values  $\delta_{12} - k_{12}v_x$  and  $\delta_{23} - k_{23}v_y$ , which provides the desired dependence of  $F_y$  on  $v_x$ . The maximum in  $F_y$  is expected for atoms with velocity

( $v_x = \delta_{12}/k_{12}$ ,  $v_y = \delta_{23}/k_{23}$ ), where each step is resonant.

The force can be calculated by using density matrices and the Ehrenfest theorem [14]. Given the levels participating in the interaction, the Rabi frequencies  $\Omega_{ij}$ , detuning values  $\delta_{ij}$ , the wavevectors  $\mathbf{k}_{ij}$  of the lasers driving the transitions, and the decay parameters of the excited states ( $\Gamma_{ji}$  is the decay rate via  $|j\rangle \rightarrow |i\rangle$ ; the total width of state  $|j\rangle$  is  $\Gamma_j = \sum_{i<j} \Gamma_{ji}$ ), we first solve the optical Bloch equations to find the density matrix  $\hat{\rho}$ . Then, the force is given by  $\mathbf{F} = \langle -\nabla_r \hat{H} \rangle = -\text{Tr}(\hat{\rho} \nabla_r \hat{H})$ , where  $\nabla_r = \hat{\mathbf{x}}\partial/\partial x + \hat{\mathbf{y}}\partial/\partial y$  [14]. The density matrix depends on the Doppler shifted detuning values  $\delta_{ij} - \mathbf{k}_{ij} \cdot \mathbf{v}$ , which provides velocity dependence of the force [14].

It should be emphasized that the ideas presented here for constructing synthetic Lorentz forces are general and potentially applicable to various atomic species. For concreteness, the ideas are presented for  $^{87}\text{Rb}$  atoms using experimentally relevant atomic states and transitions. The three-level system that can be used to experimentally realize the simplest scheme is presented in Fig. 1(b). The transition wavelengths are  $\lambda_{12} = 780$  nm and  $\lambda_{23} = 776$  nm [20]. The decay rate of the  $|5P_{3/2}\rangle$  hyperfine states is  $\Gamma_P = 2\pi \times 6.1$  MHz, and  $\Gamma_D = 2\pi \times 0.66$  MHz for  $|5D_{5/2}\rangle$  states; the decay pattern is  $\Gamma_{32} = \Gamma_D$ ,  $\Gamma_{31} = 0$ , and  $\Gamma_{21} = \Gamma_P$  [20]. In Fig. 1(c,d) we illustrate  $\mathbf{F}(\mathbf{v})$  for detuning values  $\delta_{12} = -0.5\Gamma_P$ ,  $\delta_{23} = 0$ , and Rabi frequencies  $\Omega_{12} = 0.12\Gamma_P$ , and  $\Omega_{13} = 0.34\Gamma_P$ . As expected, the maximum of the force  $F_y$  occurs when  $v_x = \delta_{12}/k_{12}$  and  $v_y = \delta_{23}/k_{23}$ . Interestingly,  $F_y(v_x, v_y)$  has the shape of a mountain ridge peaked at  $\delta_{12} + \delta_{23} - k_{12}v_x - k_{23}v_y = 0$ . This is a consequence of the fact that the intermediate state  $|2\rangle$  is much broader than state  $|3\rangle$ . For the two-step absorption to be effective, Doppler shifted detuning of the first photon should roughly be  $|\delta_{12} - k_{12}v_x| < \Gamma_2$ , and total detuning  $|\delta_{12} + \delta_{23} - k_{12}v_x - k_{23}v_y| < \Gamma_3$ ; since  $\Gamma_3 \ll \Gamma_2$ , the velocities satisfying these inequalities are close to the ridge line. The ridge can be shifted in the  $v_x v_y$  plane by changing the detuning values. The scheme above illustrates the main idea towards constructing the synthetic Lorentz force via the Doppler effect.

Note that the force in the  $x$ -direction is also altered for atoms with velocities at the ridge. The presence of second step transition  $|2\rangle \rightarrow |3\rangle$  changes the populations of the three levels, which affects the rate of first step transition  $|1\rangle \rightarrow |2\rangle$  and hence  $F_x$ . It should be noted that deformations of the ridge can arise for larger Rabi frequencies due to the Autler-Townes effect [21].

In order to provide a general framework for our scheme we Taylor expand the force up to the linear term in velocity:

$$\begin{aligned} \begin{bmatrix} F_x \\ F_y \end{bmatrix} &= \begin{bmatrix} F_{x0} \\ F_{y0} \end{bmatrix} + \begin{bmatrix} \alpha_{xx} & 0 \\ 0 & \alpha_{yy} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} 0 & \alpha_{xy} \\ \alpha_{yx} & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} \\ &= \mathbf{F}_0 + \mathbf{F}_D(\mathbf{v}) + \mathbf{F}_{SL}(\mathbf{v}). \end{aligned} \quad (1)$$

This form is often an excellent approximation for  $\mathbf{F}(\mathbf{v})$

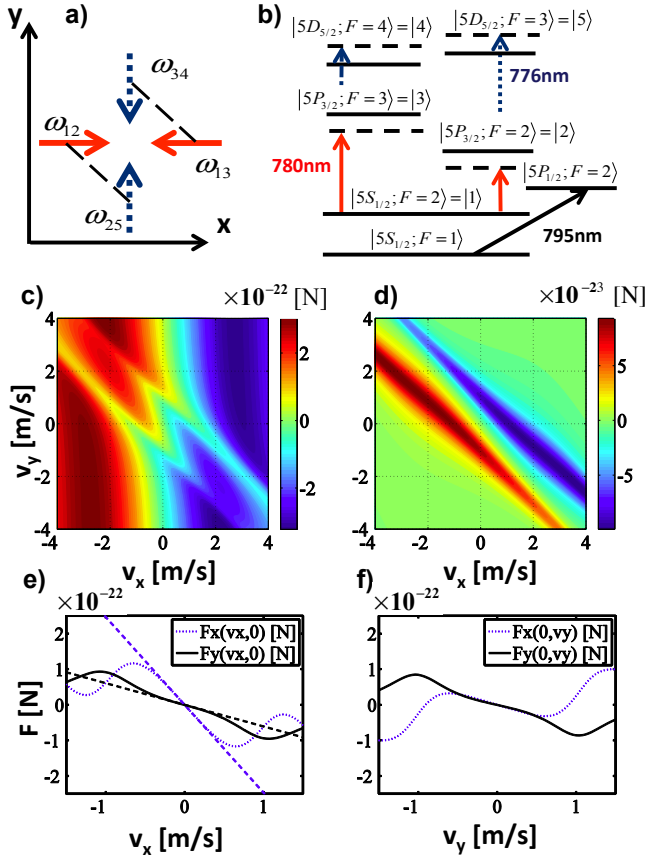


FIG. 2. The five-level scheme. (a) The symmetric configuration of lasers for creating the synthetic Lorentz force, and (b) its realization with hyperfine  $^{87}\text{Rb}$  levels (b). Density plots of  $F_x$  (c), and  $F_y$  (d) as a function of velocity. Cross sections  $F_x(v_x, 0)$  and  $F_y(v_x, 0)$  (e), and  $F_x(0, v_y)$  and  $F_y(0, v_y)$  (f). See text for details.

because of the small velocities of cold atoms. The third term  $\mathbf{F}_{SL}(\mathbf{v})$  is a general form of the synthetic Lorentz force with components perpendicular to the velocity components:  $F_{SL,x} = \alpha_{xy}v_y$ ,  $F_{SL,y} = \alpha_{yx}v_x$  [22]. The force on a standing atom is  $\mathbf{F}_0$ ; the components of the standard Doppler force are  $F_{D,x} = \alpha_{xx}v_x$ , and  $F_{D,y} = \alpha_{yy}v_y$ . When  $\alpha_{xy} = -\alpha_{yx}$ ,  $\mathbf{F}_{SL}$  takes the form of the standard Lorentz force:  $\mathbf{F}_{SL} = \mathbf{v} \times \mathbf{B}^*$ , where  $\mathbf{B}^* = \alpha_{xy}\hat{\mathbf{z}}$  [22].

Let us illustrate a few force patterns  $\mathbf{F}(\mathbf{v})$  that can be achieved with our scheme. Consider a system of five-level atoms and two orthogonal pairs of counter-propagating beams depicted in Fig. 2(a). This is simply a generalization of the idea presented in Fig. 1 with a symmetric pair of two-step arms such that  $\mathbf{F}_0 = 0$ . It can be experimentally realized by using hyperfine levels of  $^{87}\text{Rb}$  depicted in Fig. 2(b); the use of re-pumper laser is mandatory since the chosen five-level system is not closed:  $|5P_{3/2}; F=2\rangle \xrightarrow{50\%} |5S_{1/2}; F=1\rangle = |0\rangle \xrightarrow{795\text{nm}} |5P_{1/2}; F=2\rangle = |6\rangle \xrightarrow{50\%} |5S_{1/2}; F=2\rangle$  (this is included in our calculations). The Rabi frequencies and detun-

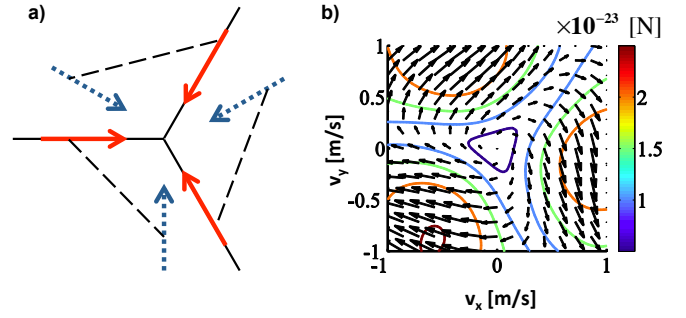


FIG. 3. The tripod configuration of the three two-step excitations arms, and the obtained force. (a) Red solid arrows depict first-step excitations (red detuned), and blue dotted arrows depict second steps (blue detuned). Black dashed lines connect beams that correspond to one arm. (b) Contour lines and length of the arrows correspond to the magnitude of the force  $|\mathbf{F}(\mathbf{v})|$ .

ing values for the transitions are  $\Omega_{12} = \Omega_{13} = 0.11\Gamma_P$ , and  $\delta_{12} = \delta_{13} = -0.5\Gamma_P$ . The pairs of beams along  $x$  are red detuned, while the pairs along  $y$  are blue detuned (by a smaller magnitude):  $\delta_{25} = \delta_{34} = 0.25\Gamma_P$ ;  $\Omega_{25} = \Omega_{34} = 0.38\Gamma_P$ . The repumper is on resonance with high intensity  $\Omega_{06} = 1.77\Gamma_P$ , in a standing wave configuration (it does not produce net force on atoms). The decay pattern is given by  $\Gamma_{21} = \Gamma_{20} = 0.5\Gamma_P$ ,  $\Gamma_{31} = \Gamma_P$ ,  $\Gamma_{43} = \Gamma_D$ ,  $\Gamma_{53} = 0.2\Gamma_D$ ,  $\Gamma_{52} = 0.8\Gamma_D$ , and  $\Gamma_{60} = \Gamma_{61} = 0.5\Gamma_P$ ; the rest of  $\Gamma_{ji} = 0$ . In Fig. 2(c,d) we show the force  $\mathbf{F}(\mathbf{v})$ . Atoms moving towards the left (right) will experience  $F_y > 0$  ( $F_y < 0$ , respectively). From Fig. 2(e,f) we see that the force depends linearly on the velocity  $v_x$  for velocities below  $\sim 0.7$  m/s (which includes essentially all atoms in a standard  $^{87}\text{Rb}$  MOT) [14]. The two ridges in  $F_y$  correspond to the pair of arms of the two-step absorption; their position and shape was explained in Fig. 1(d). By changing the detuning values, the ridges can be shifted in the  $v_x v_y$  plane, which changes the parameters  $\alpha_{ij}$  and therefore the strength of the synthetic Lorentz force.

It should be noted that because our approach is based on the Doppler effect, it usually also yields the Doppler (cooling) force  $\mathbf{F}_D$ . If for some reason this is not wanted, dissipation can be diminished (for example by using one blue and one red detuned laser in the counterpropagating configuration for the first step excitation). Moreover, the synthetic force can be made of the form  $\mathbf{v} \times \mathbf{B}^*$ : By using three arms of the two-step scheme at  $120^\circ$  [Fig. 3(a)], one can obtain the force plotted in Fig. 3(b). Two arms are identical as in Fig. 2(b), and the third arm is  $|5S_{1/2}; F=2\rangle \rightarrow |5P_{3/2}; F=1\rangle \rightarrow |5D_{5/2}; F=2\rangle$ . The Rabi frequency of the first (second) step in all arms is  $0.11\Gamma_P$  ( $0.77\Gamma_P$ ); the detuning values are  $-0.5\Gamma_P$  ( $0.25\Gamma_P$ ) for the first (second) step. Clearly the force rotates around zero in the  $v_x v_y$  plane. By fitting  $\mathbf{F}(\mathbf{v})$  to Eq. (1) we obtain  $\alpha_{xy} = -\alpha_{yx} = 0.23 \times 10^{-21}$  Ns/m, i.e.,

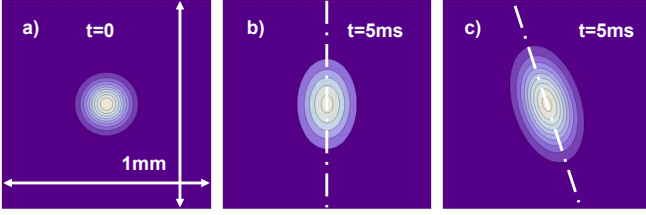


FIG. 4. Dynamics of the shape of the cloud during expansion in the presence of synthetic Lorentz force  $\mathbf{F}_{SL}(\mathbf{v})$  and/or Doppler force  $\mathbf{F}_D(\mathbf{v})$ . The force corresponds to that from Fig. 2. (a) Density of the cloud at  $t = 0$ , (b) after 5 ms of expansion under the action of  $\mathbf{F}_D(\mathbf{v})$ , (c) and after 5 ms expansion in the presence of  $\mathbf{F}_D(\mathbf{v}) + \mathbf{F}_{SL}(\mathbf{v})$ . See text for details.

$\mathbf{B}^* = \alpha_{xy}\hat{\mathbf{z}}$ ; the same force on electrons is achieved with a magnetic field of 1.5 mT. It should be emphasized that because we are using hyperfine levels of  $^{87}\text{Rb}$ , the scheme can be achieved with two CW lasers at 780 nm and 776 nm by using acoustic optical modulators (AOMs), i.e., it is experimentally viable.

The prediction of the synthetic Lorentz force is made for individual atoms, however, we should propose its signature in the CM motion and/or shape of a cold atomic cloud containing a huge number (say  $\sim 10^9$  [14]) of atoms. To this end we propose a quench-type scenario(s). First, we assume that an atomic cloud is present in the MOT, and cooled to mK- $\mu\text{K}$  temperatures. The laser fields driving the MOT have much larger Rabi frequencies than lasers producing synthetic Lorentz force. The latter will slightly heat up the cloud, but will not change its shape. Then, at  $t = 0$ , the MOT lasers and the magnetic field are suddenly turned off (it can be done within less than  $1 \mu\text{s}$  which is essentially instantaneous for this system). After  $t = 0$ , the cloud starts moving in the presence of the synthetic Lorentz and Doppler forces. We focus on dynamics in the  $xy$  plane (assume that gravity is in the  $z$  direction and does not influence observations).

We will discuss two scenarios for the force plotted in Fig. 2. First, if the cloud is given an initial velocity ( $v_x = 0.15 \text{ m/s}$ ,  $v_y = 0$ ), it will due to inertia move along  $x$ , but its CM will also move in the negative  $y$  direction due to the synthetic Lorentz force. After 10 ms the shift in  $y$  is  $\sim 0.2 \text{ mm}$ , which is observable in MOT experiments. Initial velocity can be achieved by inducing oscillations of the cloud in the MOT trap for  $t < 0$  (e.g., see [23]).

Second we discuss expansion of the cloud by employing the Fokker-Planck equation [14]:

$$\frac{\partial P(\mathbf{x}, \mathbf{v}, t)}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} P = \frac{-1}{m} \nabla_{\mathbf{v}} \cdot [(\mathbf{F}_D + \mathbf{F}_{SL})P] + \frac{D}{m^2} \nabla_{\mathbf{v}}^2 P. \quad (2)$$

Here,  $P(\mathbf{x}, \mathbf{v}, t)$  is the distribution of particles in the phase space;  $D$  is the diffusion constant, approximately given by  $D \approx (\hbar k)^2 \sum_j \rho_{jj} \Gamma_j$  [14], where  $k \approx 2\pi/780 \text{ nm}^{-1}$ ;  $\nabla_{\mathbf{v}} = \hat{\mathbf{x}}\partial/\partial v_x + \hat{\mathbf{y}}\partial/\partial v_y$ . For forces  $\mathbf{F}_D + \mathbf{F}_{SL}$  linearized in velocity (1), the Fokker-Planck

equation is solved by the ansatz:

$$P(x, y, v_x, v_y, t) = P_0 \exp\left\{-\sum_{ij=1}^4 \frac{1}{2} a_{ij}(t) \eta_i \eta_j\right\}, \quad (3)$$

where  $(\eta_1, \eta_2, \eta_3, \eta_4) = (x, y, v_x, v_y)$ ; after inserting (3) in Eq. (2), one obtains 10 coupled ordinary differential equations (ODEs) for the functions  $a_{ij}(t)$ ; 10 because  $a_{ij} = a_{ji}$  by construction. These coupled ODEs are readily solved numerically in MATHEMATICA and results are plotted in Fig. 4 for the following parameters:  $(\alpha_{xx}, \alpha_{xy}, \alpha_{yx}, \alpha_{yy}) = -(2.4, 0.60, 0.69, 0.58) \times 10^{-22} \text{ Ns/m}$ , and  $D/m_{\text{Rb}}^2 = 31 \text{ m}^2\text{s}^{-3}$ ; the initial state is  $P = P_0 \exp\{-(\mathbf{x}/x_0)^2 - (\mathbf{v}/v_0)^2\}$ , where  $x_0 = 1 \text{ mm}$ , and  $v_0 = 0.25 \text{ m/s}$ . Starting from a centrosymmetric cloud plotted in Fig. 4(a), in the presence of solely the Doppler force, the cloud expands asymmetrically [Fig. 4(b)] because  $|\alpha_{xx}| > |\alpha_{yy}|$ . The signature of the synthetic Lorentz force is the rotation of the asymmetric cloud in the  $xy$  plane during expansion [see Fig. 4(c)]. The interpretation is simple: particles moving to the left (right) are pushed up (down), as can be inferred from Fig. 2(d). There is another effect: the change in  $F_y$  for a given atomic velocity group also changes  $F_x$  for that group, as discussed above. For the parameters corresponding to Figs. 2 and 4, besides the targeted  $\alpha_{yx} < 0$ , we incidentally also obtained  $\alpha_{xy} < 0$  (for small velocities) which also contributes to rotation in the same direction.

In conclusion, we have demonstrated a scheme for creating synthetic Lorentz forces in cold classical atomic clouds, based on the Doppler effect and radiation pressure. We envision that following this scheme one could produce a tokamak type trap for cold atoms, and potentially mimic tokamak plasma. We have predicted forces of magnitude  $F/v \sim 0.23 \times 10^{-21} \text{ Ns/m}$  (corresponding to 1.5mT fields acting on electrons) in macroscopic volumes of a few  $\text{mm}^3$  and more. We envision that our concept involving two-photon absorption could be applicable in other systems (e.g., suspended nanoparticles), for developing new types of optical tweezers, or for velocity selection of atomic beams.

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\* hbuljan@phy.hr

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